AFRL-ML-WP-TP-2006-499

STATISTICAL ANALYSIS AND COMPUTER GENERATION OF SPATIALLY CORRELATED ACOUSTIC NOISE (PREPRINT)

Raina Cepel, Lori Thombs, Dominic Ho, and Steven P. Neal

MAY 2006

Approved for public release; distribution is unlimited.

STINFO COPY

MATERIALS AND MANUFACTURING DIRECTORATE
AIR FORCE RESEARCH LABORATORY
AIR FORCE MATERIEL COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OH 45433-7750

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. REPORT DATE *(DD-MM-YY)*	2. REPORT TYPE	3. DATES COVERED *(From - To)*
May 2006	Journal Article Preprint	01/01/2005 – 12/31/2005

4. TITLE AND SUBTITLE
STATISTICAL ANALYSIS AND COMPUTER GENERATION OF SPATIALLY CORRELATED ACOUSTIC NOISE (PREPRINT)

5a. CONTRACT NUMBER FA8650-04-C-5704
5b. GRANT NUMBER
5c. PROGRAM ELEMENT NUMBER 62102F

6. AUTHOR(S)
Raina Cepel and Steven P. Neal (University of Missouri-Columbia/ Mechanical and Aerospace Engineering)
Lori Thombs (University of Missouri-Columbia/Department of Statistics)
Dominic Ho (University of Missouri-Columbia/Electrical and Computer Engineering)

5d. PROJECT NUMBER 2510
5e. TASK NUMBER 00
5f. WORK UNIT NUMBER 00

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
University of Missouri-Columbia
Mechanical and Aerospace Engineering
E3401 Lafferre Hall
Columbia, MO 65211

University of Missouri-Columbia
Department of Statistics
Columbia, MO 65211

University of Missouri-Columbia
Electrical and Computer Engineering
Columbia, MO 65211

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Materials and Manufacturing Directorate
Air Force Research Laboratory
Air Force Materiel Command
Wright-Patterson AFB, OH 45433-7750

10. SPONSORING/MONITORING AGENCY ACRONYM(S)
AFRL-ML-WP

11. SPONSORING/MONITORING AGENCY REPORT NUMBER(S)
AFRL-ML-WP-TP-2006-499

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES
This work, resulting in whole or in part from Department of the Air Force contract FA8650-04-C-5704, has been submitted to Springer Netherlands for publication in the Journal of Nondestructive Evaluation. PAO Case Number: AFRL/WS 06-1405, 1 Jun 2006.

14. ABSTRACT
In ultrasonic NDE, simulation studies can play an important role in complimenting experimental validation of techniques under development. The utility of such simulations depends, in part, on the degree to which the simulated defect and noise signals are representative of the measured signals. In this paper, we describe an approach for generating simulated acoustic noise with a spatial correlation coefficient distribution and maximum extreme value (MEV) distribution which matches those distributions for measured acoustic noise. The procedure for generating noise signals is outlined for a line scan and for a raster scan. The basic approach forces the correlation of neighboring signals to the desired correlation by creating each signal as the sum of appropriately scaled neighboring signals plus a new random signal. For the line scan where each interior position has only two neighbors, this process is done sequentially without iteration. For the raster scan where each interior point has four nearest neighbors, iteration is required to simultaneously achieve the desired correlations with row and column neighbors. The MEV distribution is controlled in an outer iterative loop with the shape and position of the distribution dictated by spectral content of the noise signals and by controlling the signal energy, respectively. Results are shown which demonstrate the effectiveness of the approach. With this approach, a limited number of measured signals can be used to establish the correlation coefficient and MEV distributions which drive the computer generation of a large number of simulated acoustic noise signals.

15. SUBJECT TERMS ultrasonics, acoustic noise, correlation

16. SECURITY CLASSIFICATION OF: a. REPORT	b. ABSTRACT	c. THIS PAGE	17. LIMITATION OF ABSTRACT:	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON (Monitor)
Unclassified	Unclassified	Unclassified	SAR	46	Gary Steffes **19b. TELEPHONE NUMBER** *(Include Area Code)* N/A

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std. Z39-18

Statistical Analysis and Computer Generation of Spatially Correlated Acoustic Noise

Raina Cepel,[1] Lori Thombs,[2] Dominic Ho,[3] and Steven P. Neal[1]

[1] Mechanical and Aerospace Engineering
[2] Department of Statistics
[3] Electrical and Computer Engineering

University of Missouri-Columbia, Columbia, MO 65211

ABSTRACT

In ultrasonic NDE, simulation studies can play an important role in complimenting experimental validation of techniques under development. The utility of such simulations depends, in part, on the degree to which the simulated defect and noise signals are representative of the measured signals. In this paper, we describe an approach for generating simulated acoustic noise with a spatial correlation coefficient distribution and maximum extreme value (MEV) distribution which matches those distributions for measured acoustic noise. The procedure for generating noise signals is outlined for a line scan and for a raster scan. The basic approach forces the correlation of neighboring signals to the desired correlation by creating each signal as the sum of appropriately scaled neighboring signals plus a new random signal. For the line scan where each interior position has only two neighbors, this process is done sequentially without iteration. For the raster scan where each interior point has four nearest neighbors, iteration is required to simultaneously achieve the desired correlations with row and column neighbors. The MEV distribution is controlled in an outer iterative loop with the shape and position of the distribution dictated by spectral content of the noise signals and by controlling the signal energy, respectively. Results are shown which demonstrate the effectiveness of the approach. With this approach, a limited number of measured signals can be used to establish the correlation coefficient and MEV distributions which drive the computer generation of a large number of simulated acoustic noise signals.

Keywords: ultrasonics, acoustic noise, correlation

I. INTRODUCTION

Simulation studies are routinely used in ultrasonic nondestructive evaluation (NDE) to complement experimental studies during the development of new inspection approaches. Simulated A-scans typically include a target signal (e.g., a flaw or weld plane signal) plus noise. The noise of interest lies between the front and back surface reflections in an A-scan and is comprised of electronic plus acoustic noise. In certain cases, the degree of correlation between A-scans can have a significant influence on detection.[1-4] These correlations can be quantified in terms of the spatial cross-correlation, at zero lag, between gated A-scans.[4,5] Uncorrelated noise A-scans are easily generated using a normal random number generator with filtering used to achieve the desired frequency content. Margetan et al. have gone a step further using an independent scatterer model to generate simulated grain noise A-scans with correlations between A-scans controlled to some degree through use of an experimentally determined spatial correlation length parameter.[2] Using their approach, the average, maximum, and standard deviation of gated peak-values are also controlled. In the current paper, we describe an approach for generating simulated acoustic noise with a correlation coefficient distribution and maximum extreme value (MEV) distribution (i.e, the gated peak-value distribution) which matches those distributions determined for measured acoustic noise.

The motivation for this project finds its origin in research associated with kissing bond detection for inertial welding of two stainless steel pieces. Kissing bonds can be very difficult and expensive to fabricate in a controlled fashion, providing motivation for simulation studies which utilize simulated noise signals with realistic correlation

coefficient distributions. The basic approach being developed relies on the correlation coefficients between adjacent A-scans to detect low signal-to-noise ratio (SNR) kissing bond signals. This approach finds its genesis in the work of Nagy and Adler on this same problem.(6) Without going into the analysis details, suffice it to say that an inspection approach can be formulated which relies heavily on the comparison between correlation coefficient distributions associated with backscattered signals from the weld being inspected and from a known set of acceptable welds.(7) More recently the correlation approach has been extended to crack detection in both pulse/echo and pitch/catch.(8)

Computer generation of correlated random variables with a desired mean correlation coefficient is straightforward. For example, the i^{th} random sample can be generated based on the $i^{th}-1$ random sample as follows: (5)

$$x(i) = a(i) + bx(i-1) \tag{1}$$

where b is an adjustable scale factor, $a(i)$ is the output of a random number generator, and the x's are the computer generated samples of the random variable, **x**. The single scale factor can be adjusted to yield the desired mean correlation coefficient value, but, in general, the scale factor cannot be adjusted such that desired correlation coefficient mean and distribution (width and shape) are achieved. The natural relationship between the mean and shape dictate that as b is increased, correlation values are forced closer to the limiting value of 1.0, the distribution breadth decreases, and the distribution becomes increasingly skewed with a fat lower tail. The primary task addressed in this paper is to extend the approach represented in Eq. (1) so that vectors of random numbers (simulated

A-scans) can be generated that show the desired distribution of correlation coefficient values. These simulated signals will also be forced to the desired MEV distribution.

Simulation of data with specified correlation structure arises in many fields. In the physical sciences, scientists often use Shewhart Control Charts to monitor a process. In this setting, data are a time-indexed series of averages or counts taken at regularly spaced time intervals. Padgett, Thombs and Padgett [(9)] present a method to generate such one-dimensional data with specified mean and variance structure so that the performance of such charts can be studied via simulation. In this paper, the generated data are two dimensional, with emphasis on both the correlation within and between series, as well and the maximum extreme value distribution.

Two dimensional spatial time series data with correlation structure is common in many other areas, including meteorology, hydrology and ecological and environmental studies. In the area of ecology and wildlife studies, observations such as animal counts are typically observed at irregularly-spaced locations. Neighboring observations are likely to be correlated, and the paper by Brooker [(10)] represents one of the first attempts to generate such data.

In spatial statistics, point patterns are observed on variables such as temperature and rainfall, so that the data often include a (third) component, time. See Cressie [(11)] for more information on the statistical aspects of fitting models to such data. A recent contribution by Kyriakidis et al.[(12)] proposes an algorithm for generating spatio-temporal precipitation data, with emphasis on preserving the distribution of the original data set. Data are both space-indexed (e.g., longitude and latitude) and time indexed.

The methodology presented in the current paper is distinct from these related approaches for generating dependent data in that both the correlation and the MEV distribution are controlled. The paper proceeds by first establishing representative correlation coefficient and MEV distributions associated with measured backscattered noise. The methodology for generation of simulated acoustic noise with the desired correlation coefficient and MEV distributions is then described for a line scan and for a raster scan. Results are presented which validate the approach for the experimentally established correlation coefficient and MEV distributions. The paper closes with a brief summary section.

II. MEASUREMENT PROCEDURE

Backscattered grain noise signals were measured and used as a basis for calculating associated correlation coefficient and MEV distributions. The ultrasonic measurement system used in these measurements consists of a water tank filled with degassed tap water at approximately 19^{o}C, a three dimensional scanning bridge that holds a transducer, a pulser-receiver unit, and a 12 bit data acquisition card with a sample rate of 100 MS/s. A dedicated PC collects data from the acquisition card and controls the motor controller that moves the scanning bridge. A separate PC is employed for data analysis.

The transducer used to make measurements was a focused ½” transducer with a 10 MHz center frequency and a 4” focal length. The settings on the pulser-receiver and the data acquisition card were typically set at values such that the front and back surface reflections were blown off the screen in order to enable proper digitization of the grain

noise. In each measurement position, 64 signals were taken and averaged together in order to reduce electronic noise.

The sample used in all measurements was a stainless steel plate with the dimensions 10.1 x 5 x 1.9 cm. A leveling plate was used to ensure that the specimen was aligned with the transducer's scan plane. The transducer was normalized in relation to the front face of the sample. Data was taken at a single water path such that the focal point of the transducer would be approximately at the sample mid-plane. Signals were measured on a 3.2 x 2.4 cm grid with 0.5 mm between measurement positions.

After the data was collected, pre-processing was done on the raw signals. First, the front surface reflections of each signal were aligned with one another. Then all of the signals were averaged together in order to identify any non-random component associated with the front surface reflections. This mean signal was subtracted from each of the individual signals so that spatial correlations between adjacent noise signals could be calculated with minimal influence from front surface reflection ringing.(2) Finally, each signal was gated to extract a time window of 51 points (0.50 μs) with the gate positioned to correspond closely with the location in the sample where the transducer was focused.

III. STATISTICAL ANALYSIS OF MEASURED NOISE

A. Maximum Extreme Value Distributions

Vectors of data representing maximum extreme values (maximum absolute values) for the measured noise were extracted from the gated signals. Figure 1 shows 2 histograms based on this MEV data. The lower histogram is for all measured signals, that is, for a measurement position spacing of 0.5 mm. The upper histogram is for every

3rd signal, corresponding to a measurement spacing of 1.5 mm. The histograms are fundamentally the same with the number of observations being the only significant difference. An extreme value distribution based on the absolute value of the difference between min and max values could have been utilized instead.(2)

B. Spatial Correlation Coefficient Distributions

The correlation of interest is the spatial cross-correlation calculated at zero lag between gated A-scans measured at adjacent measurement positions. For discussion purposes, consider an *N x M* scan with the A-scan (each *T* points long) written into the matrix $\mathbf{x} = x(i, j, t)\ i = 1, N\ \ j = 1, M\ \ t = 0, T-1$ (see Fig. 2). Calculation of the correlation coefficient between A-scans is then given by the following equation where $\rho = \rho(i, j, t_i, t_f, \delta_r, \delta_c)$:

$$\rho = \frac{\sum_{t=t_i}^{t_f}\left[x(i,j,t) - m_x(i,j)\right]\left[x(i+\delta_c, j+\delta_r, t) - m_x(i+\delta_c, j+\delta_r)\right]}{\sqrt{\sum_{t=t_i}^{t_f}\left[x(i,j,t) - m_x(i,j)\right]^2}\sqrt{\sum_{t=t_i}^{t_f}\left[x(i+\delta_c, j+\delta_r, t) - m_x(i+\delta_c, j+\delta_r)\right]^2}} \quad (2)$$

In Eq. (2), the summation range in the time-domain defines the portion of the signal (the time window or gate) of interest, m is the mean value calculated over the gate, and δ is a spatial shift parameter. Throughout the paper, t is used as a discrete index referring to the temporal direction. With $\delta_r = 1\ \ \delta_c = 0$, row correlations are calculated between adjacent A-scan, that is, A-scans measured at the j^{th} and $j^{th} + 1$ positions in the i^{th} scan row. Similarly, with $\delta_r = 0\ \ \delta_c = 1$, column correlations can be calculated. Correlations

for all possible adjacent signal combinations in an *N x M* raster scan can be established using a computation loop over *i* and *j* with the spatial shift applied sequentially to *i* and *j*.

In order to define desired distributions of spatial correlation coefficients for simulated noise, conditional correlation coefficient distributions were established based on the pre-processed measured noise signals. In this section, we begin by considering the overall distribution of correlation coefficients. Conditional distributions are addressed below. The first step was to calculate the correlation coefficient (Eq. 2) between each signal and its row and column neighbors (see Fig. 2) and write these values into a single vector. This calculation was done separately for three measurement spacings: 0.5 mm spacing (every signal), 1 mm spacing (every 2nd signal), and 2 mm spacing (every 4^{th} signal). The correlation coefficient histograms for these measurement spacings are shown in Fig. 3. The influence of measurement spacing on both the mean and the shape of the correlation coefficient distribution is apparent. Conditional correlation coefficient distributions show the same basic characteristics as these overall distributions.

Probability density function fits to the distributions were considered. The normal and gamma distributions were found to be the two common distributions that yielded the best fit to the correlation distributions. The normal distribution works best when the average correlation value is near zero and the distribution is symmetric; however, as is apparent from the figure, the distribution tends to show a suppressed peak and fatter tails than the normal distribution. The gamma distribution works well for skewed distributions with higher average correlations. In some cases, a standard distribution which shows reasonable fit to the histogram cannot be found; however, an interpolation of the histogram can be used to approximate the actual distribution of the correlation

coefficient values. The interpolation approach utilizes the Matlab routine randsample to randomly choose a bin and then randomly select a value from each bin based on a linear probability density function defined between the edges of each bin.(13) Superimposed on each histogram in Figure 3 is a normal or gamma distribution fit along with the interpolation fit.

In general, correlation coefficient values are not randomly distributed in space. That is, the correlation coefficient between a given pair of signals depends on the correlation coefficients between surrounding pairs of signals. As an example, Fig. 4 shows the variation in correlation values along a line scan with relatively low (upper graph) and high (lower graph) measurement spacing. As demonstrated in the figure, the dependence of correlation values on adjacent values decreases with increasing measurement spacing. A graphical example of a conditional histogram is shown by the stem-plot in Fig. 5. The overall distribution is given by stems terminating in light circles, and the conditional distribution, assuming an adjacent correlation value of 0.58, is given by the stems terminating in dark circles.

IV. GENERATION OF SPATIALLY CORRELATED NOISE

A. Generation of Spatially Uncorrelated Acoustic Noise

A number of equally effective approaches could be taken to generate uncorrelated acoustic noise. As depicted in Fig. 6, the basic steps in the process as implemented here are as follows: 1) using a standard normal random number generator, create a time-domain white noise signal, T points long, for each (simulated) measurement position; 2) Fourier transform each signal to the frequency domain; 3) filter each signal using a filter (with unit energy) that will force the resultant noise to have the same average power

spectrum as the measured noise; and 4) inverse Fourier transform each signal back to the time-domain to yield A-scans which are spatially uncorrelated with appropriate frequency content. Note that the correlated noise creation step described below involves sums of scaled A-scans. This process changes the average power spectrum from the desired spectrum associated with these initial spatially uncorrelated signals. An iterative correction approach will be outlined which brings the average power spectrum back to the desired shape while still achieving the desired correlation distribution was implemented.

B. One Dimensional Generation of Spatially Correlated Noise

We begin by establishing the approach for a line scan and then expand the procedure for application to an *xy* raster scan in the next section. Starting at the noise measurement stage, assume that a line scan is performed, acoustic noise signals are measured and pre-processed, the average power spectrum is estimated, and the overall and conditional correlation coefficient distributions are established. The line scan of measured signals are written in a two-dimensional matrix, denoted $\mathbf{x}_m$, with average power spectral density function estimate and associated magnitude spectrum given by $\left|\mathbf{X}_m(f)\right|^2$ and $\left|\mathbf{X}_m(f)\right|$, respectively.

For discussion purposes, assume that an N position line scan is to be simulated. $N-1$ correlation coefficients are randomly generated with the first value coming from the overall distribution and subsequent values drawn from conditional correlation coefficient distributions. These $N-1$ correlation coefficients, denoted $\tilde{\rho}(i)$ $i=2,N$, will dictate the correlations between simulated noise signals.

Clarification of the overall and conditional correlation coefficient distributions is in order. The overall distribution is the probability density function associated with the probability $P(\rho(i))$, that is, for the entire set of correlation coefficients given that these correlation coefficients are not independent. Before addressing conditional distributions, recall the notation: $\rho(i)$ gives the correlation between $x(i,t)$ and $x(i-1,t)$; $\rho(i-1)$ gives the correlation between $x(i-1,t)$ and $x(i-2,t)$; and $\rho(i+1)$ gives the correlation between $x(i+1,t)$ and $x(i,t)$. Considering only nearest neighbors, the conditional probability density function for $\rho(i)$ is associated with the probability: $P(\rho(i)|\,\rho(i-1) \text{ and } \rho(i+1))$. In practice, during the sequential generation of correlated signals, the correlation coefficient $\rho(i)$ is drawn from the distribution associated with $P(\rho(i)|\,\rho(i-1))$ since $\rho(i+1)$ does not yet exist.

The process of creating correlated noise signals involves several steps. We begin by describing the procedure used to force the desired correlation between to two given signals. This approach is then incorporated into an iterative procedure used to simultaneously match the desired correlation distribution and average power spectrum.

Figure 7 gives a cartoon representation of the notation and some of the steps involved in creating a simulated line scan of spatially correlated signals. Using the procedure described in the previous section, the first step is to create a set of N uncorrelated acoustic noise signals, represented by the matrix $\mathbf{a} = a(i,t)$ $i = 1, N$ $t = 0, T-1$. The output matrix, i.e., the set of correlated signals, is denoted $\mathbf{x} = x(i,t)$ $i = 1, N$ $t = 0, T-1$. The next step is to set $x(1,t)$ equal to $a(1,t)$, that is, $x(1,t) = a(1,t)$ $t = 0, T-1$. Throughout the remainder of the paper, all operations will

be implicitly carried out over the range of *t*, for example, $x(1,t) = a(1,t) \Rightarrow x(1,t) = a(1,t) \;\; t = 0, T-1$. The second signal, $x(2,t)$, is then to be determined so that the correlation between $x(2,t)$ and $x(1,t)$ is equal to the desired correlation, $\tilde{\rho}(2)$. Consistent with approach of Eq. (1), the method used to calculate $x(2,t)$ is to add $b(2)x(1,t)$ to the uncorrelated signal $a(2,t)$: $x(2,t) = a(1,t) + b(2)x(1,t)$. The key is to find the value of $b(2)$ which forces the correlation between $x(1,t)$ and $x(2,t)$ to be equal to $\tilde{\rho}(2)$. This process will be repeated for each signal, that is:

$$x(i,t) = a(i,t) + b(i)x(i-1,t) \qquad i = 2, N \tag{3}$$

Note that $x(i,t)$ will be appropriately correlated with both of its neighbors, $x(i-1,t)$ and $x(i+1,t)$, since the process forces correlations between $x(i,t)$ and $x(i-1,t)$ and between $x(i+1,t)$ and $x(i,t)$.

The process for establishing the scale factor, $b(i)$, can be described as follows. We begin by using the correlation coefficient between $a(i,t)$ and $x(i-1,t)$ to calculate the inherent similarity between the two signals that will be used to create the output signal.

$$\rho(i) = \frac{\sum_{t=0}^{T-1}\left(a(i,t) - m_a(i)\right)\left(x(i-1,t) - m_x(i)\right)}{\sqrt{\sum_{t=0}^{T-1}\left(a(i,t) - m_a(i)\right)^2}\sqrt{\sum_{t=0}^{T-1}\left(x(i-1,t) - m_x(i)\right)^2}} \qquad i = 2, N \tag{4}$$

In order to simplify the notation, the mean subtraction step will be implicit in future correlation equations. For ease of calculations, the following substitutions will be used.

$$
\begin{aligned}
r_i &= \sum_{t=0}^{T-1} a(i,t)x(i-1,t) \\
s_i &= \sum_{t=0}^{T-1} a(i,t)^2 \\
z_i &= \sum_{t=0}^{T-1} x(i-1,t)^2 \\
\rho(i) &= \frac{r_i}{\sqrt{s_i z_i}}
\end{aligned}
\tag{5}
$$

The goal is to force the correlation between $x(i,t)$ and $x(i-1,t)$ to be equal to $\tilde{\rho}(i)$. This correlation can be calculated as follows.

$$
\tilde{\rho}(i) = \frac{\sum_{t=0}^{T-1} x(i,t)x(i-1,t)}{\sqrt{\sum_{t=0}^{T-1} x(i,t)^2}\sqrt{\sum_{t=0}^{T-1} x(i-1,t)^2}}
\tag{6}
$$

To relate this correlation to the scale factor, $b(i)$, we proceed with manipulations of Eq. (3). First, both sides of Eq. (3) are multiplied by $x(i-1,t)$ to give the top equality in Eq. (7). Second, both sides of Eq. (3) are squared to give the middle equality in Eq. (7). This equation is then multiplied by $\sum x(i-1,t)^2$ to yield the lower equality in Eq. (7). Each equality in Eq. (7) is written in terms of the parameters defined in Eq. (5).

$$\sum_{t=0}^{T-1} x(i,t)x(i-1,t) = \sum_{t=0}^{T-1} a(i,t)x(i-1,t) + b(i)\sum_{t=0}^{T-1} x(i-1,t)^2 = r_i + b(i)z_i$$

$$\sum_{t=0}^{T-1} x(i,t)^2 = \sum_{t=0}^{T-1} a(i,t)^2 + 2b(i)\sum_{t=0}^{T-1} a(i,t)x(i-1,t) + b^2(i)\sum_{t=0}^{T-1} x(i-1,t)^2$$

$$= s_i + 2b(i)r_i + b^2(i)z_i \tag{7}$$

$$\sum_{t=0}^{T-1} x(i,t)^2 \sum_{t=0}^{T-1} x(i-1,t)^2 = z_i\left(s_i + 2b(i)r_i + b^2(i)z_i\right) = s_i z_i + 2b(i)r_i z_i + b^2(i){z_i}^2$$

Equation (6) can now be re-written as follows.

$$\tilde{\rho}(i) = \frac{\sum_{t=0}^{T-1} x(i,t)x(i-1,t)}{\sqrt{\sum_{t=0}^{T-1} x(i,t)^2 \sum_{t=0}^{T-1} x(i-1,t)^2}} = \frac{r_i + b(i)z_i}{\sqrt{s_i z_i + 2b(i)r_i z_i + b^2(i){z_i}^2}} \tag{8}$$

To obtain the desired correlation, the correct scale factor must be chosen by solving Eq. (8) for $b(i)$. To solve for $b(i)$, both sides of Eq. (8) are squared, the resultant equation is rearranged into a quadratic polynomial, and solved as follows.

$$\tilde{\rho}^2(i) = \frac{r_i^2 + 2b(i)r_i z_i + b^2(i)z_i^2}{s_i z_i + 2b(i)r_i z_i + b^2(i)z_i^2}$$

$$\left(z_i^2(1-\tilde{\rho}^2(i))\right)b^2(i) + \left(2z_i r_i(1-\tilde{\rho}^2(i))\right)b(i) + \left(r_i^2 - s_i z_i \tilde{\rho}^2(i)\right) = 0 \tag{9}$$

$$b(i) = \frac{-r_i}{z_i} \pm \sqrt{\frac{\tilde{\rho}^2(i)(s_i z_i - r_i^2)}{(1-\tilde{\rho}^2(i))}}$$

If the desired correlation is positive ($\tilde{\rho}(i) > 0$), the maximum value of $b(i)$ is chosen; if the desired correlation is negative ($\tilde{\rho}(i) < 0$), the minimum value of $b(i)$ is chosen. In the discussion given below, the argument of the radical is shown to be non-negative.

At this point, the correlation part of the line scan problem is solved. With $b(i)$ calculated as given in Eq. (9) and following the procedure outlined above culminating in Eq. (3), simulated acoustic noise signals can be created with a correlation coefficient distribution which matches the desired distribution.

C. Matching the Maximum Extreme Value Distribution

A secondary point of emphasis was to force the simulated noise to match the MEV distribution for the measured noise. When using a normal random number generator approach to create simulated acoustic noise signals, the mean of the MEV distribution is controlled by the standard deviation of the random number generator. As discussed below, the mean of the MEV distribution was handled at the end of the correlated noise creation step by scaling each signal to control the average signal energy. The shape of the MEV distribution depends on the correlation between points in each A-scan; in other words, the shape is controlled by the autocorrelation function or average power spectrum of the noise. As described above, the initial set of uncorrelated noise signals

($a(i,t)$ $i = 1, N$) are filtered to force an average magnitude spectrum which matches that of the measured noise. The summation process depicted in Eq. (3) alters the average spectrum for the output correlated signals ($x(i,t)$ $i = 1, N$). Unfortunately, correcting the average spectrum of the correlated signals back to the desired spectrum would alters signal correlations and associated correlation coefficient distribution; thus, motivating the implementation of an iterative approach. The goal is to filter the noise going in to the correlation process so that the noise coming out of the correlation process will have the desired average power spectrum and, therefore, the desired MEV distribution shape.

Additional notation is needed to describe the iterative approach. The output of the correlation process for the k^{th} iteration will be denoted $\mathbf{x}_k = x_k(i,t)$ $i = 1, N$ with average power spectrum and magnitude spectrum given by $|\mathbf{X}_k(f)|^2$ and $|\mathbf{X}_k(f)|$, respectively. The filters will be denoted $\mathbf{F}_k(f)$ with the initial filter, $\mathbf{F}_1(f)$, based directly in the magnitude spectrum for the measured noise: $\mathbf{F}_1(f) = |\mathbf{X}_m(f)|$. The initial filter, $\mathbf{F}_1(f)$, is the filter used in creating the uncorrelated signals ($\mathbf{a} = a(i,t)$ $i = 1, N$). The goal is then to make the output spectrum, $|\mathbf{X}_k(f)|$, equal to the desired spectrum, $|\mathbf{X}_m(f)|$, to within some acceptable error. Defining E_x to be the sum of squared errors between $|\mathbf{X}_k(f)|$ and $|\mathbf{X}_m(f)|$ over some frequency range and ε_x to be the acceptable error level, the iterative process will continue until $E_x \le \varepsilon_x$. The filter is updated after each iteration based on the ratio of the desired and output spectra: $|\mathbf{X}_m(f)| / |\mathbf{X}_k(f)|$.

For a line scan, the iterative approach used to achieve the desired correlation coefficient distribution and frequency spectrum can be summarized in algorithm form as follows:

0. generate a set of white noise signals : $\mathbf{a}_w = a_w(i,t) \;\; i = 1, N$

1. apply the filter, $\mathbf{F}_k(f)$, to the white noise to yield : $\mathbf{a} = a(i,t) \;\; i = 1, N$

2. calculate $\mathbf{x}_k$: $\;\; x_k(1,t) = a(1,t) \quad x_k(i,t) = x_k(i-1,t) + b(i)a(i,t) \quad i = 2, N$

3. calculate $|\mathbf{X}_k(f)|$ and the error : $E_x = \sum_{f_i}^{f_f} \left(|\mathbf{X}_m(f)| - |\mathbf{X}_k(f)|\right)^2$

4. if $E_x \le \varepsilon_x$; $\mathbf{x} = \mathbf{x}_k$; stop;

5. if $E_x > \varepsilon_x$; calculate an updated filter : $\; k = k + 1 \quad \mathbf{F}_k(f) = \mathbf{F}_{k-1}(f)\dfrac{|\mathbf{X}_m(f)|}{|\mathbf{X}_{k-1}(f)|}$

6. return to step 1.

Empirical evidence shows that the final filter shape is strongly dependent on the desired magnitude spectrum and weakly dependent on the desired conditional correlation coefficient distributions. As such, for a given desired magnitude spectrum, we generally run through the iterative process one time to establish the appropriate filter shape. This filter is then used for all additional runs to generate correlated noise signals, regardless of the desired output correlation coefficient distributions.

Finally, the signals are scaled to achieve the desired mean MEV, that is, the mean MEV associated with the measured acoustic noise signals, denoted m_{mev}. As indicated earlier, when generating uncorrelated noise signals ($\mathbf{a} = a(i,t) \;\; i = 1, N$) using a standard normal random number generator with $\sigma = 1$ will force the mean MEV for the signals in $\mathbf{a}$ to unity. The mean MEV for the correlated noise signals ($\mathbf{x} = x(i,t) \;\; i = 1, N$) can also

be forced to 1 by making the energy of each correlated noise signal equal to the energy of the associated uncorrelated signal. The desired mean MEV can then achieved by scaling by the mean MEV of the measured noise, denoted m_{mev}. This can be done in the computational loop or as a post-processing step as follows: $\mathbf{x} = \mathbf{x}(\sigma_a / \sigma_x) m_{mev}$ where σ_a and σ_x are the standard deviation of the uncorrelated and correlated noise, respectively. Note that scaling the signals does not change the correlation coefficient distribution since the correlation coefficient is scale independent.

We close the section by showing that $b(i)$ is purely real and by considering some special cases. To prove that $b(i)$ will never be imaginary, the value under the square root must be shown to be non-negative. There are three values under the square root $\tilde{\rho}^2(i)$, $(1-\tilde{\rho}^2(i))$, and $(s_i z_i - r_i^2)$. Since ρ is always between –1 and 1, the first two quantities are clearly greater than or equal to zero. To show that the third quantity is always positive, consider the following manipulations of Eq. (6?), noting that $\tilde{\rho}^2(i) \geq 0$.

$$\rho(i) = \frac{r_i}{\sqrt{s_i z_i}} \quad \Rightarrow \quad r_i^2 = \tilde{\rho}^2(i) s_i z_i \quad \Rightarrow \quad r_i^2 \leq s_i z_i \qquad (10)$$

A few special cases should be examined. The first case is where the desired correlation is ± 1. From Eq. (9), it can be seen that setting the desired correlation to ± 1 makes the denominator under the square root equal to 0, forcing $b(i)$ to $\pm\infty$. With reference to Eq. (3), this makes sense because $\tilde{\rho}(i) = \pm 1$ implies $x(i,t) = \pm x(i-1,t)$ which

can only be approximated with $b(i)=\pm\infty$ ($a(i,t)\neq 0$). If the desired correlation were ± 1, $x(i,t)$ should simply be set to $\pm x(i-1,t)$.

A second special case would arise if the original correlation (Eq. (5)) were equal to ± 1. For this case, $r_i^2 = s_i z_i$ and the numerator under the radical in Eq. (9) goes to zero. This eliminates the influence of the desired correlation value, $\tilde{\rho}(i)$, and results in $b(i)=-r_i/z_i$. We note from Eq. (8?) that $b(i)=-r_i/z_i \Rightarrow r_i=-b(i)z_i$ will merely set the output correlation to 0. However, when dealing with randomly generated noise, it is exceedingly unlikely that we will come across two signals that have a correlation of 1. If this case were to occur, the problem could be solved by generating a new $a(i,t)$.

D. Two Dimensional Generation of Spatially Correlated Noise

The procedure for generating correlated A-scans which simulate an *xy* raster scan follows directly from the procedure for a line scan. Figure 8 defines some of the notation used in this section. For the two dimensional case, the correlated signals are created in an inner iterative loop with a second outer iterative loop used to simultaneous satisfy the correlation distribution and frequency content requirements. We again begin by assuming that a set of data is taken by measuring backscattered signals at *N x M* equally spaced measurement positions (see Fig. 1). The correlation coefficients between neighboring signals can be used to define the overall and conditional correlation coefficient distributions. These distributions can be used to generate $N(M-1)$ correlation coefficients between signals in a row, denoted $\tilde{\rho}_r(i,j)$, and $(N-1)M$ correlation coefficients between signals in a column, denoted $\tilde{\rho}_c(i,j)$. These are the desired correlation values that will be used in determining the scale factors, similarly

denoted $b_r(i, j)$ and $b_c(i, j)$. As with the line scan case, desired correlation coefficients can only be drawn from distributions conditioned on correlation coefficient values between previously generated signals.

The procedure again starts with the generation of a set of uncorrelated acoustic noise signals, now written into a thee-dimensional matrix: $\mathbf{a} = a(i, j, t)\ i = 1, N\ \ j = 1, M$. The first row and the first column are treated as line scans. The first output signal, $x(1,1,t)$, is set equal to $a(1,1,t)$. The procedure described above for a line scan is then used to find the scale factors, $b_r(1, j)\ \ j = 2, M$ and $b_c(i,1)\ \ i = 2, N$, and the associated simulated signals, $x(1, j, t)\ \ j = 2, M$ and $x(i,1,t)\ \ i = 2, N$.

The remainder of the correlated signals are created in an iterative fashion starting with $x(2,2,t)$. Each new signal is simultaneously forced toward the desired correlation with two neighboring signals. In general notation (see Fig. 12), $x(i, j, t)$ is iteratively forced toward the desired correlation with $x(i-1, j, t)$ and $x(i, j-1, t)$ by using the sum of appropriately scaled versions of these two signals plus $a(i, j, t)$.

To generate the signal $x(i, j, t)$, the algorithmic loop can be summarized as follows:

1. Calculate the scale factors required to force the desired correlation between $x(i, j, t)$ and $x(i-1, j, t)$, denoted $\tilde{\rho}_c(i, j)$, and between $x(i, j, t)$ and $x(i, j-1, t)$, denoted $\tilde{\rho}_r(i, j)$:

$$b_c(i,j) = \frac{-r_{cij}}{z_{cij}} \pm \sqrt{\frac{\tilde{\rho}_c^2(i,j)(s_{cij}z_{cij} - r_{cij}^2)}{(1-\tilde{\rho}_c^2(i,j))}}$$

$$b_r(i,j) = \frac{-r_{rij}}{z_{rij}} \pm \sqrt{\frac{\tilde{\rho}_r^2(i,j)(s_{rij}z_{rij} - r_{rij}^2)}{(1-\tilde{\rho}_r^2(i,j))}} \qquad (11)$$

2. Calculate the output signal using both scale factors and all three signals:

$$x(i,j,t) = a(i,j,t) + b_r(i,j,)x(i,j-1,t) + b_c(i,j)x(i-1,j,t) \qquad (12)$$

3. Calculated the actual correlation coefficient between $x(i,j,t)$ and $x(i-1,j,t)$,denoted $\bar{\rho}_c(i,j)$, and between $x(i,j,t)$ and $x(i,j-1,t)$, denoted $\bar{\rho}_r(i,j)$,

$$\bar{\rho}_c(i,j) = \frac{\sum_{t=0}^{T-1} x(i,j,t)x(i-1,j,t)}{\sqrt{\sum_{t=0}^{T-1} x(i,j,t)^2}\sqrt{\sum_{t=0}^{T-1} x(i-1,j,t)^2}}$$

$$\bar{\rho}_r(i,j) = \frac{\sum_{t=0}^{T-1} x(i,j,t)x(i,j-1,t)}{\sqrt{\sum_{t=0}^{T-1} x(i,j,t)^2}\sqrt{\sum_{t=0}^{T-1} x(i,j-1,t)^2}} \qquad (13)$$

4. Calculate the correlation coefficient error and compare with the acceptable error level, ε_ρ:

$$\delta\rho_c(i,j) = \left|\tilde{\rho}_c(i,j) - \bar{\rho}_c(i,j)\right| \qquad \delta\rho_r(i,j) = \left|\tilde{\rho}_r(i,j) - \bar{\rho}_r(i,j)\right| \tag{14}$$

If $\delta\rho_c(i,j) \le \varepsilon_\rho$ *and* $\delta\rho_r(i,j) \le \varepsilon_\rho$ then stop; else go to step 5.

5. Use the current output signal as a new starting signal: $a(i,j,t) = x(i,j,t)$. Return to step 1.

The loop is repeated until $NM - (N+M)$ correlated acoustic noise signals have been generated. Note that $x(i,j,t)$ is correlated with each of its four neighbors since $x(i,j,t)$ is calculated based on its correlation with $x(i-1,j,t)$ and $x(i,j-1,t)$, and $x(i+1,j,t)$ and $x(i,j+1,t)$ are calculated based on their correlations with $x(i,j,t)$ (see Fig. 8).

Finally, the MEV distribution shape and position are addressed. As with the line scan, an outer iterative loop is used to force the generated signals to have the desired frequency content, and thus the MEV distribution shape, while maintaining the desired correlation coefficient distribution. This iterative process follows directly from the steps outlined for a line scan in the previous section with only notational changes required to account for three-dimensional rather than two-dimensional matrices. The position of the MEV distribution for the simulated signals is again dealt with by scaling of the simulated signals, following the approach outlined for the line scan: $\mathbf{x} = \mathbf{x}(\sigma_a/\sigma_x)m_{mev}$.

IV. RESULTS

A. Implementation

The MEV distributions (Fig. 1) and conditional correlation coefficient distributions associated with the overall distributions shown in Fig. 3 were used as the desired distributions to demonstrate implementation of the noise generation approach. A periodogram approach was used to estimate the average magnitude spectrum of the measured noise for use in creating acoustic noise signals with the desired MEV distribution shape. Desired correlation coefficients were drawn from interpolation-based conditional probability density functions which were based on conditional histograms constructed from the measured noise. Spatially correlated acoustic noise A-scans, each 51 points long, were generated to simulate a 50 x 50 raster scan at a digitization rate of 100 MS/s (10 ns/point).

B. Results

We begin by showing an example of the evolution of correlation coefficients and signals toward the desired result for the middle distribution shown in Fig. 3. The two dashed lines in Fig. 9 show examples of the how the actual correlation values, $\bar{\rho}_r(i, j)$ and $\bar{\rho}_c(i, j)$, move toward the desired values, $\tilde{\rho}_r(i, j)$ and $\tilde{\rho}_c(i, j)$, with each iteration. Figure 10 shows how one uncorrelated noise signal, $a(i, j)$ (solid lines, upper graphs), can start at an arbitrary correlation coefficients with its neighboring signals (dashed lines in the upper and lower graphs) and then after iteration evolve to a correlated noise signal, $x(i, j)$ (solid lines, lower graphs), showing the desired correlations with its neighbors.

The method outlined for creating correlated noise was tested for each of the overall correlation coefficient distributions shown in Fig. 3. Given the stochastic nature

of this problem, each time a set of correlated noise signals is generated, the resultant histograms represent one realization of noise generation process. In this section, we show example histograms for one realization of the process, and we give chi-squared analysis results that quantify the average agreement between the desired and output histograms.

Desired and actual histograms are given in Fig. 11 for overall correlation coefficient distributions and in Fig. 12 for the MEV distributions. The histograms in each figure show the desired correlation distributions (Fig. 3) and MEV distributions (Fig. 1). The star data points superimposed on the histograms represent the bin values for the histograms determined from the simulated noise. For each distribution, signals were generated to simulate a 50 x 50 scan, yielding 2500 signals, and 4900 nearest-neighbor correlations. The success of the approach is witnessed by the agreement between the histograms for measured and created noise signals. Again note that each set of simulated signals will give a slightly different correlation coefficient histograms and MEV distributions.

The results were also evaluated quantitatively using a chi-squared test. The test is based on the agreement between histogram bin values associated with the generated and measured noise, with the measured-noise based histograms representing the desired values. Using the vector h_c and h_c to hold the bin values for the calculated and measured noise, respectively, the chi squared error between the two histograms can be written as follows:

$$\chi^2 = \sum_1^k \frac{(h_m - h_c)^2}{h_m} \tag{15}$$

where k represents the number of bins. This statistic was looked up in a standard chi squared table with $k-1$ degrees of freedom. The resulting p-value is the probability that the same error would result if both histograms were indeed from the same distribution. Thus if we pick a cutoff value, such as 0.05, any p value above this cutoff is deemed acceptable.

This process was done on 10 sets of 20 x 20 of simulated noise. For the MEV histogram shown in Fig. 1, the average p-value was 0.325, with a standard deviation of 0.233. For the overall correlation histogram, the average p-value was 0.557, with a standard deviation of 0.340. For the conditional correlation histogram, the average p-value was 0.220 with a standard deviation of 0.404. These quantitative results indicate that on the average for MEV, overall correlation, and conditional correlation distribution, the distributions for the simulated noise are consistent with the desired distributions based on the measured noise.

V. SUMMARY

An approach has been outlined for generating simulated acoustic noise with a correlation coefficient distribution and MEV distribution which matches those distributions for measured acoustic noise. With this approach, a limited number of measured signals can be used to establish the correlation coefficient and MEV distributions which drive the computer generation of a large number of simulated acoustic noise signals. Signals simulated in this manner are being used in the development of a scale independent correlation based approach to defect detection.[7,8]

ACKNOWLEDGEMENTS

This work was supported by the Center for Aerospace Manufacturing Technology (CAMT) at the University of Missouri-Rolla funded under Air Force Research Laboratory Contract FA8650-04-

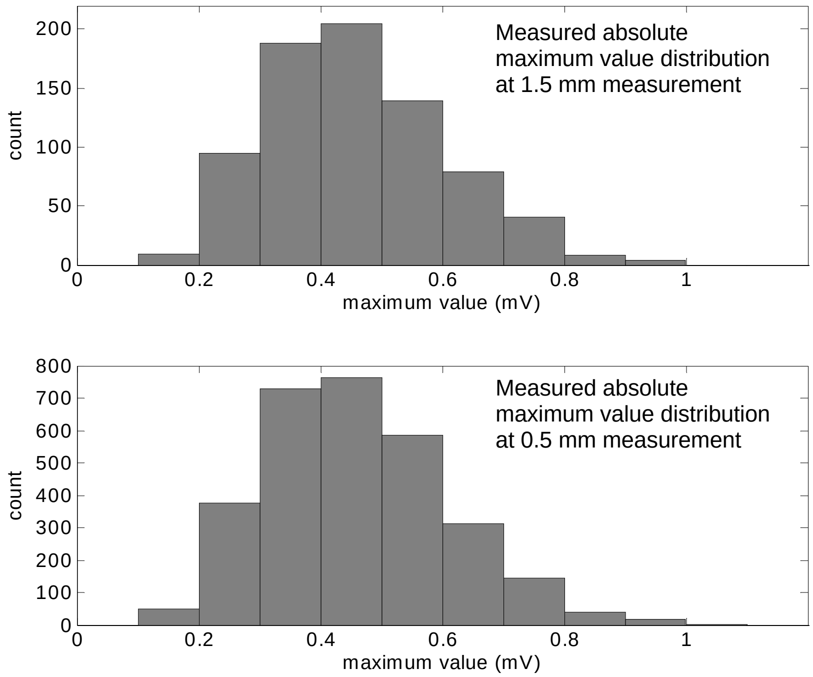
C-5704 and by Boeing under PO Z40791. Previous support for development of the correlation approach was provided by Honeywell Federal Manufacturing and Technology and by the National Science Foundation (NSF CMS 9610189). The authors would like to acknowledge early development work on correlation analysis by previous postdocs and students including Dr. Mark D. Russell, Dr. Terence P. Lerch, and Brett Rinker. The authors would also like to acknowledge the tremendous support from personnel at Honeywell FM&T including Eric Jamieson, Jose A. Samayoa, Jason P. Miller and Brett Rinker. In particular, the encouragement by Eric Jamieson to extent the correlation approach to localized flaw detection was pivotal. Many thanks are also due to Dr. Zoughi from the University of Missouri-Rolla and Dr. Palmer from the Boeing Company Phantom Works for their technical guidance and leadership.

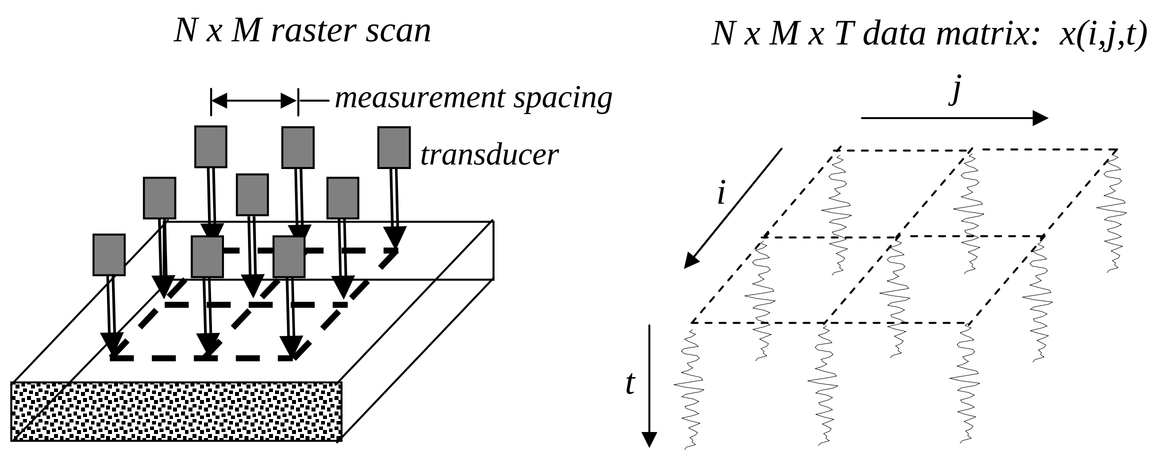
REFERENCES

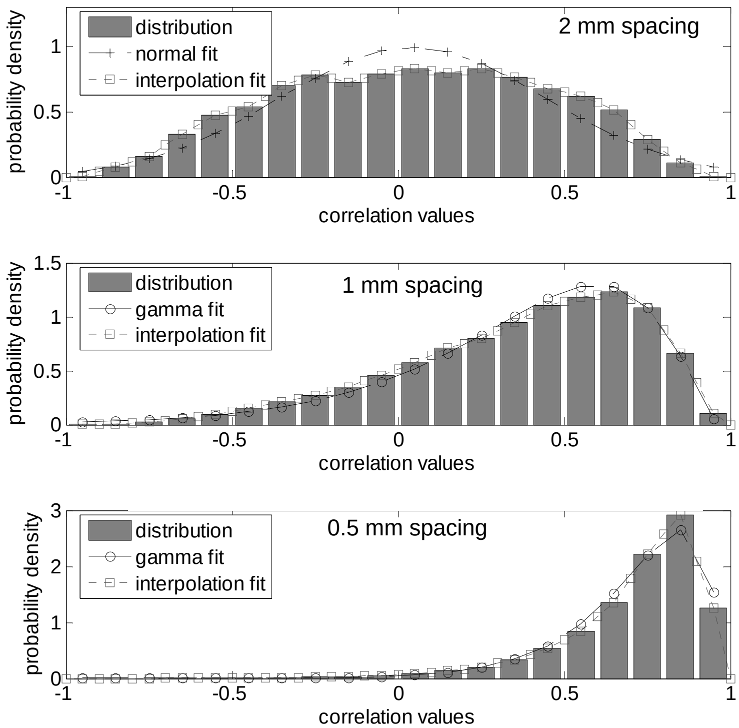
1. R. B. Thompson, L. Yu, and F. J. Margetan, "A formal theory for the spatial correlation of backscattered ultrasonic grain noise", *Review of Progress in Quantitative Nondestructive Evaluation*, Vol. 24, edited by D. E. Chimenti and D. O. Thompson, AIP, Melville, 2005, pp. 1292-1299.
2. F. J. Margetan, E. Nieters, L. Brasche, L. Yu, A. Degtyar, H. Wasan, M. Keller, and A. Kinney, "Fundamental Studies of Titanium Forging Materials - Engine Titanium Consortium II", FAA William J. Hughes Technical Center, Atlantic City, N. J., Report number DOT/FAA/AR-05/22 (June 2005).
3. A. Dogandzic and N. Eua-Anant, "Defect detection in correlated noise", *Review of Progress in Quantitative Nondestructive Evaluation*, Vol. 23A, edited by D. E. Chimenti and D. O. Thompson, AIP, Melville, 2004, pp. 628-635.
4. W. F. Walker, "The significance of correlation in ultrasound signal processing", *Proc. SPIE Vol. 4325, Medical Imaging: Ultrasonic Imaging and Signal Processing*, edited by Michael F. Insana and K. Kirk Shung, 2001, pp. 159-171.
5. J. S. Bendat and A. G. Piersol, *Engineering Applications of Correlation and Spectral Analysis*, J. Wiley, New York, p. 46, 1993.
6. Nagy, P. B. and Adler, L, "Ultrasonic NDE of Solid State Bonds: Inertia and Friction Welds", *J. Nondestructive Eval*. **7(3/4)**, 199-215, (1988).
7. B. A. Rinker, E. E. Jamieson, J. A. Samayoa, T. G. Abeln, T. P. Lerch, and S. P. Neal, "Detection of weak interface signals for same material bond/weld inspection", *Review of Progress in Quantitative Nondestructive Evaluation*, Vol. 22B, edited by D. E. Chimenti and D. O. Thompson, AIP, Melville, 2003, pp. 1080-1087.
8. R. Cepel, D. O. Palmer, Jr., and S. P. Neal, "A preliminary study of ultrasonic detection of cracks through thick composite doublers", *Proceedings of the* 9^{th} *Joint FAA/DOD/NASA Conference on Aging Aircraft*, Atlanta, GA, March 6-9, 2006.

9. C. S. Padgett, L. A. Thombs, and W. J. Padgett, “On the α-risks for Shewhart control charts", *Communications in Statistics, Part B -- Simulation and Computation*, **21**, 1125-1147, (1992).
10. P. I. Brooker, “Simulation of spatially correlated data in two dimensions", *Mathematics and Computers in Simulation*, **27**, 155-157, (1985).
11. N. Cressie, *Statistics for Spatial Data*, J. Wiley, New York, (1991).
12. P. C. Kyriakidis, N. L. Miller, and J. Kim, “A Spatial Time Series framework for simulation daily precipitation at regional scales,” *Journal of Hydrology*, **297**, 236-255, (2004).
13. MATLAB version 7.0.0. The MathWorks, Inc., Natick, Massachusetts, (2004).

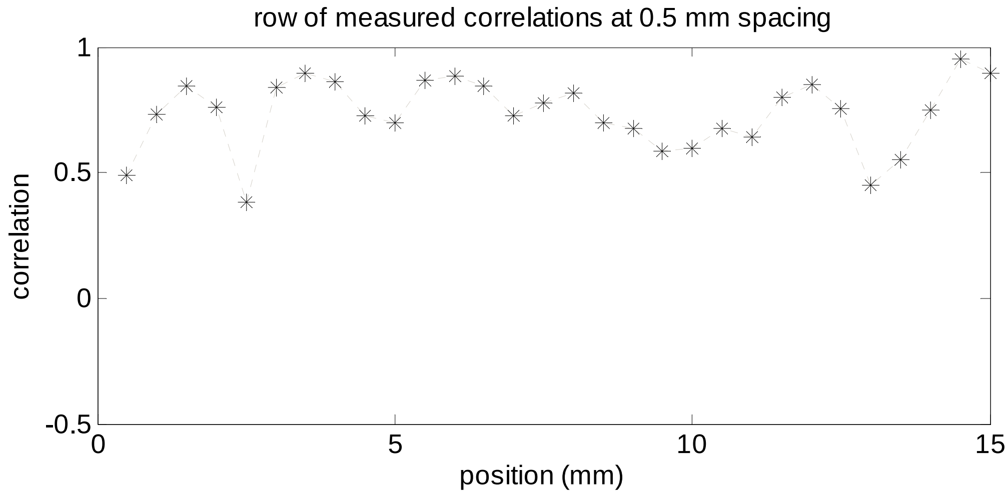
1. List of Figures

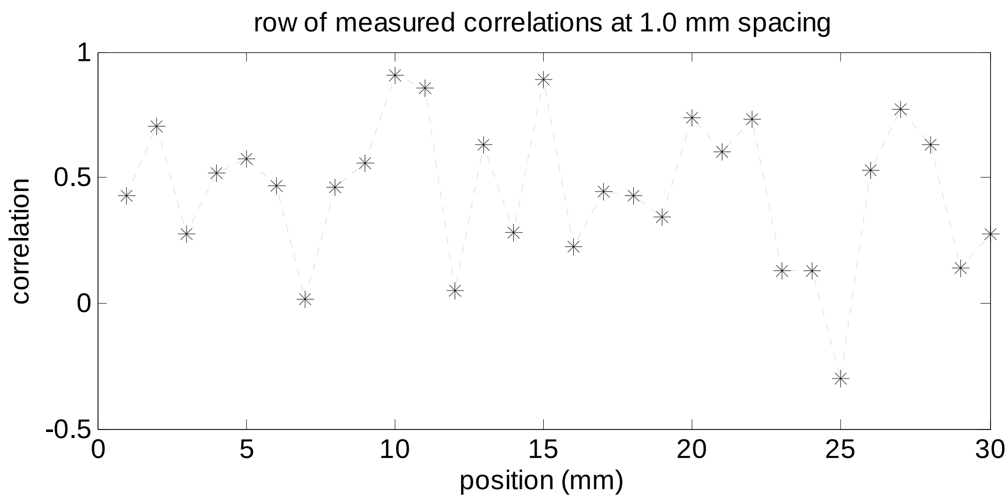

Measured absolute
maximum value distribution
at 1.5 mm measurement
count
0
50
100
150
200
0
0.2
0.4
0.6
0.8
1
maximum value (mV)
Measured absolute
maximum value distribution
at 0.5 mm measurement
count
0
100
200
300
400
500
600
700
800
0
0.2
0.4
0.6
0.8
1
maximum value (mV)

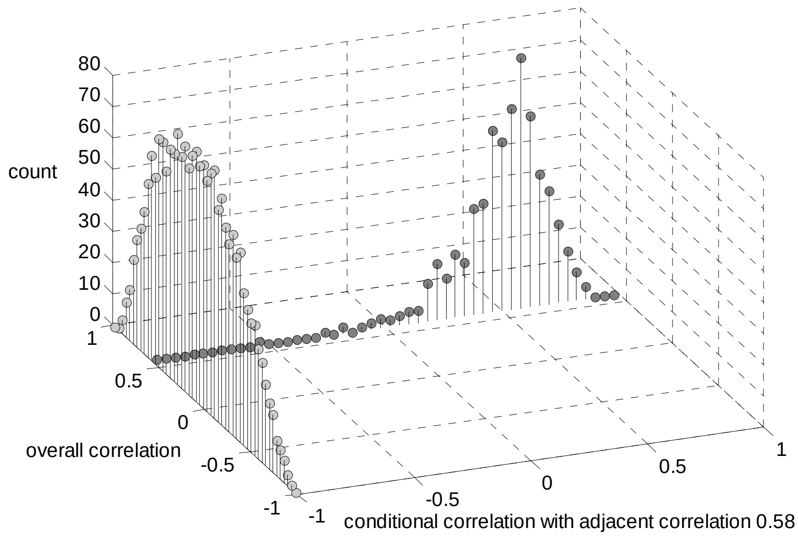

N x M raster scan
measurement spacing
transducer
N x M x T data matrix: x(i,j,t)
j
i
t



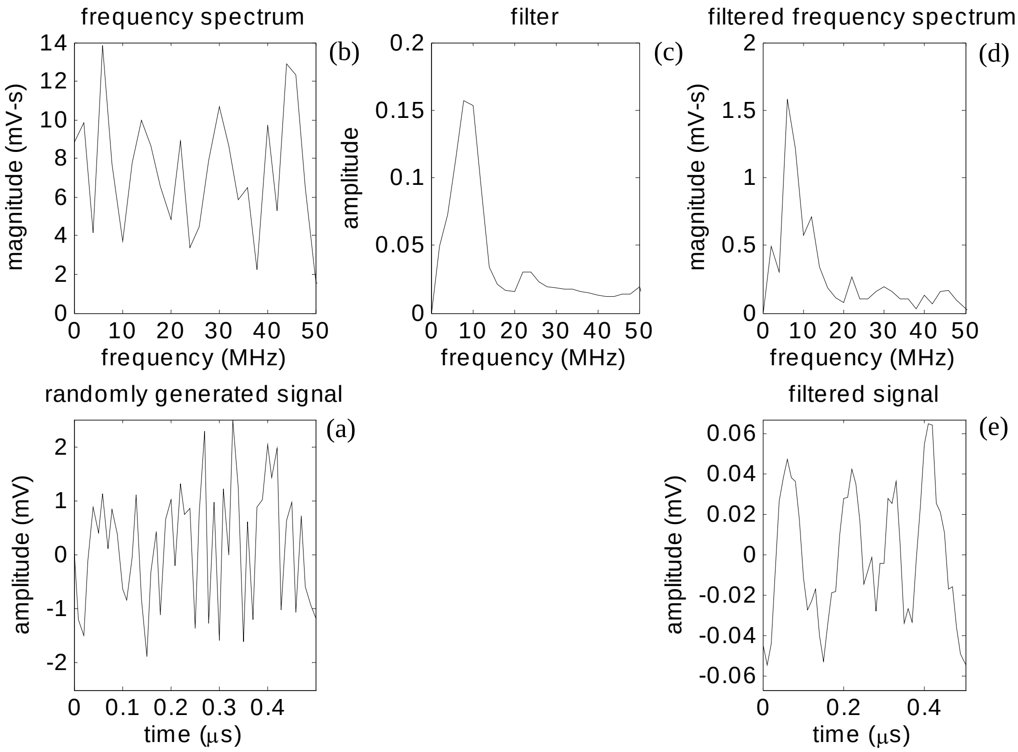
distribution
normal fit
interpolation fit
2 mm spacing
probability density
correlation values
distribution
gamma fit
interpolation fit
1 mm spacing
probability density
correlation values
distribution
gamma fit
interpolation fit
0.5 mm spacing
probability density
correlation values

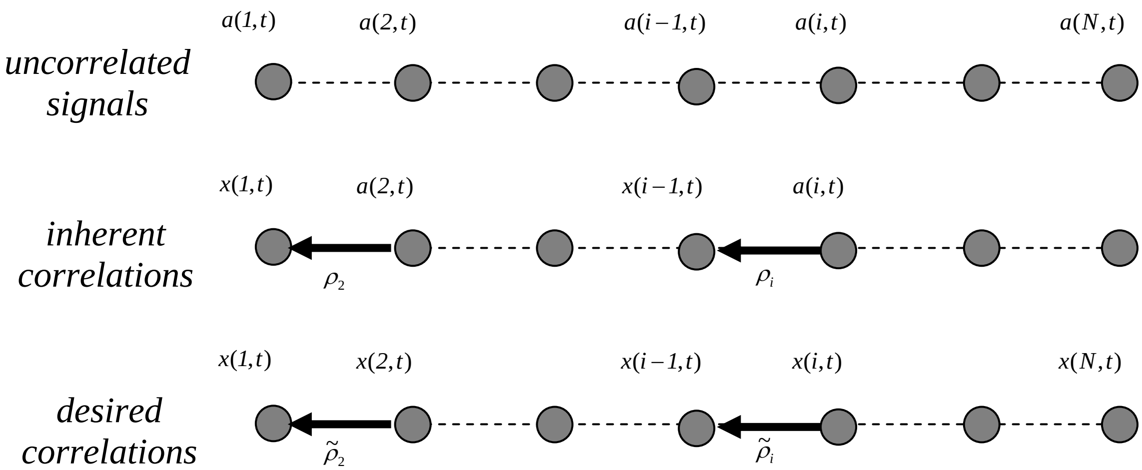

row of measured correlations at 0.5 mm spacing
correlation
position (mm)
1
0.5
0
-0.5
0
5
10
15

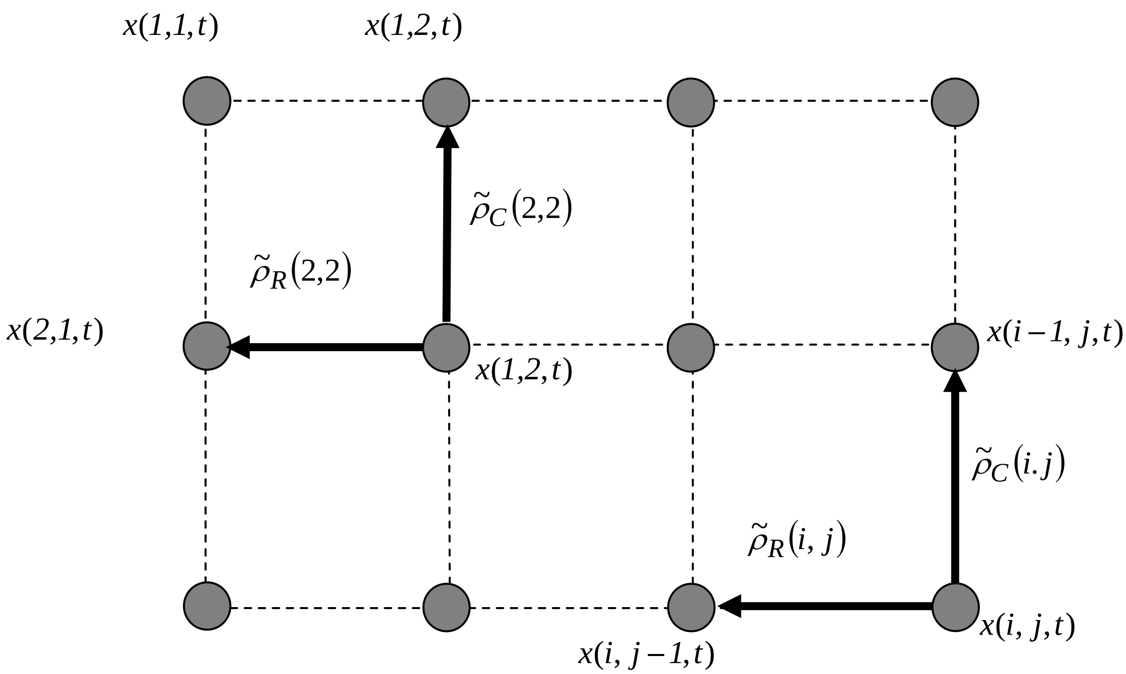

row of measured correlations at 1.0 mm spacing
correlation
position (mm)
1
0.5
0
-0.5
0
5
10
15
20
25
30



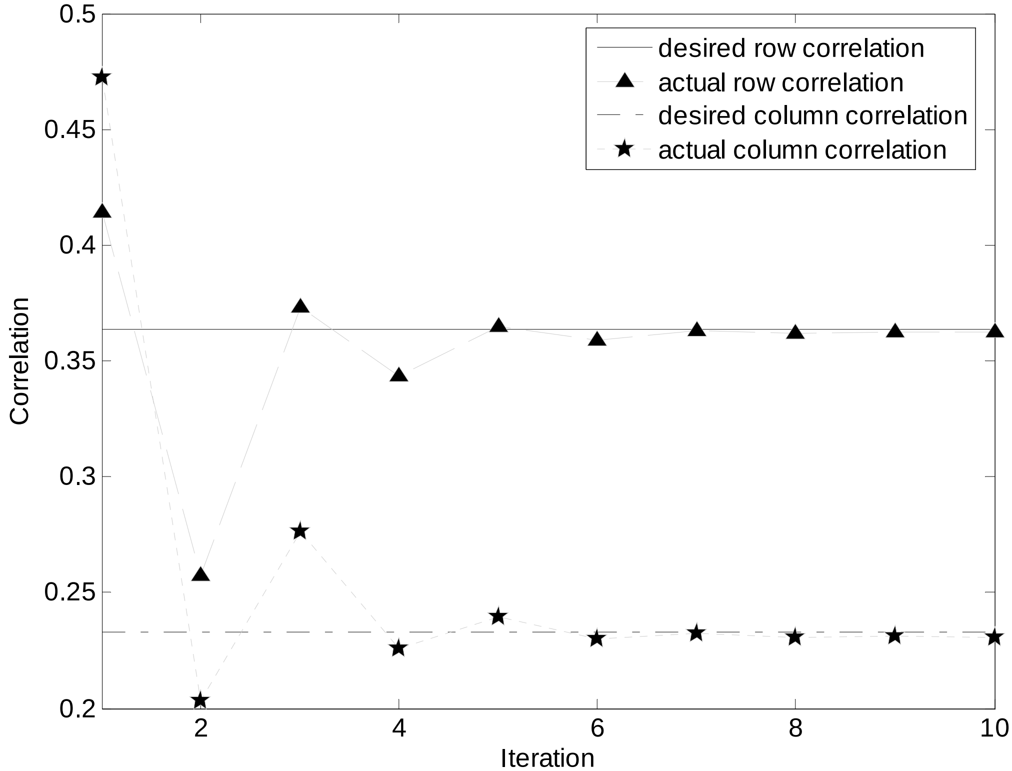
count
80
70
60
50
40
30
20
10
0
1
0.5
0
overall correlation
-0.5
-1
-1
-0.5
0
0.5
1
conditional correlation with adjacent correlation 0.58


frequency spectrum
(b)
magnitude (mV-s)
frequency (MHz)
filter
(c)
amplitude
frequency (MHz)
filtered frequency spectrum
(d)
magnitude (mV-s)
frequency (MHz)
randomly generated signal
(a)
amplitude (mV)
time (μs)
filtered signal
(e)
amplitude (mV)
time (μs)

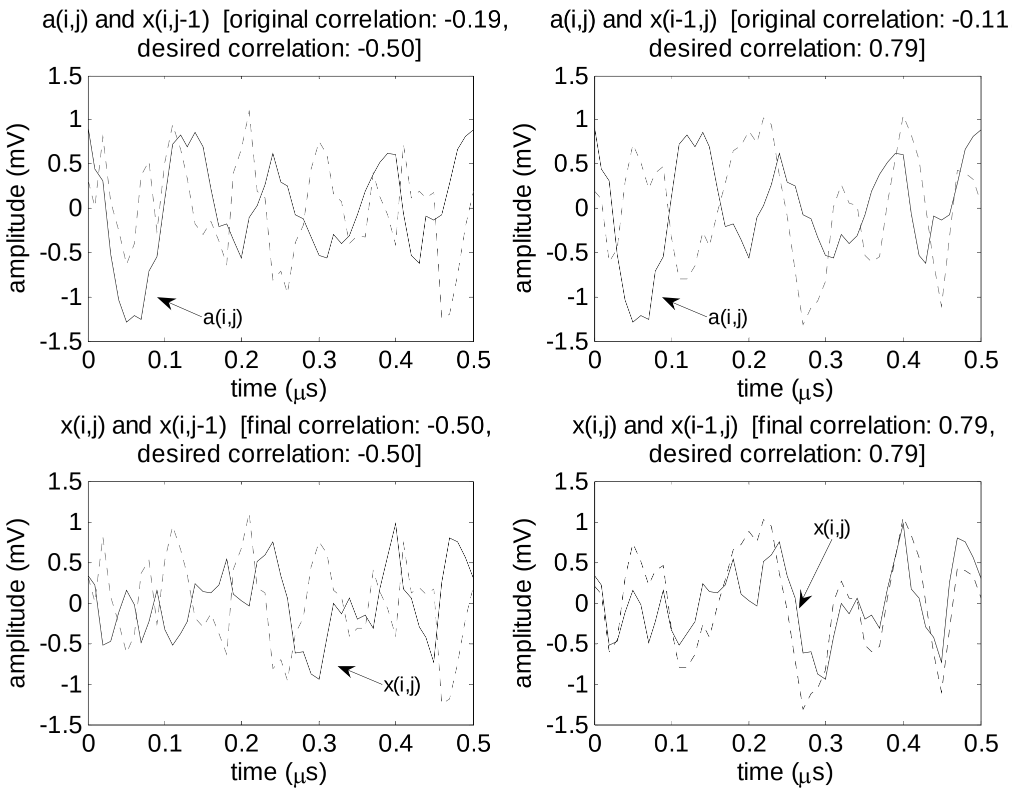

uncorrelated signals
a(1,t)
a(2,t)
a(i−1,t)
a(i,t)
a(N,t)
inherent correlations
x(1,t)
a(2,t)
x(i−1,t)
a(i,t)
ρ₂
ρᵢ
desired correlations
x(1,t)
x(2,t)
x(i−1,t)
x(i,t)
x(N,t)

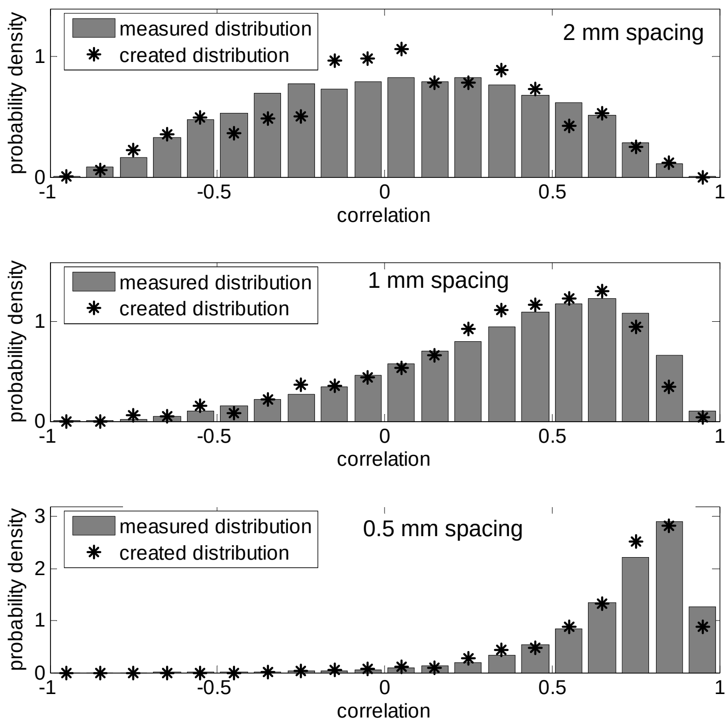


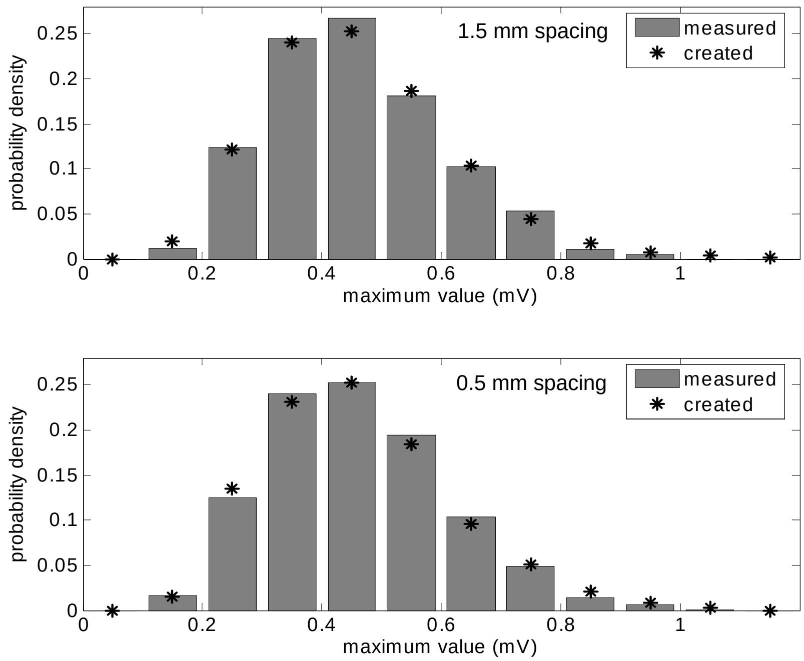
$x(1,1,t)$
$x(1,2,t)$
$\tilde{\rho}_C(2,2)$
$\tilde{\rho}_R(2,2)$
$x(2,1,t)$
$x(1,2,t)$
$x(i-1,j,t)$
$\tilde{\rho}_C(i.j)$
$\tilde{\rho}_R(i,j)$
$x(i,j,t)$
$x(i,j-1,t)$



desired row correlation
actual row correlation
desired column correlation
actual column correlation
Correlation
Iteration
0.5
0.45
0.4
0.35
0.3
0.25
0.2
2
4
6
8
10


a(i,j) and x(i,j-1) [original correlation: -0.19, desired correlation: -0.50]
a(i,j) and x(i-1,j) [original correlation: -0.11, desired correlation: 0.79]
x(i,j) and x(i,j-1) [final correlation: -0.50, desired correlation: -0.50]
x(i,j) and x(i-1,j) [final correlation: 0.79, desired correlation: 0.79]
amplitude (mV)
time (μs)
a(i,j)
x(i,j)


measured distribution
created distribution
2 mm spacing
probability density
correlation
1 mm spacing
0.5 mm spacing


1.5 mm spacing
measured
created
probability density
maximum value (mV)
0.5 mm spacing
measured
created
probability density
maximum value (mV)